

# Review Set #3



"How close to the truth to you want to come, sir?"

		Truth about the population	
		$H_0$ true	$H_a$ true
Decision based on sample	Reject $H_0$	Type I error	Correct decision
	Accept $H_0$	Correct decision	Type II error

## Tests of Hypotheses

- (Statistical) Hypothesis: an assertion concerning one or more populations.
- In statistics, there are only two states of the world:
  - $H_0$  : "equals" (null hypothesis)
  - $H_1$  : \_\_\_\_\_ (alternate hypothesis)

- Examples:

$$H_0 : \mu = 17$$

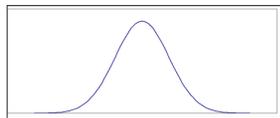
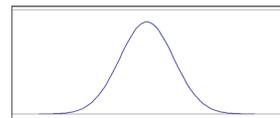
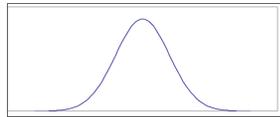
$$H_1 : \mu \neq 17$$

$$H_0 : \mu = 8$$

$$H_1 : \mu > 8$$

$$H_0 : p = 0.5$$

$$H_1 : p < 0.5$$



# Inference

# Things to Remember

**If the question asks:** Is there any statistical evidence...**Run some type of inference test**

- a. Hypothesis Test for Means (1 or 2 samples or paired t-test)
- b. Hypothesis Test for Proportions (1 or 2 samples)
- c. Chi-squared test (Goodness of Fit or Independence or Homogeneity)
  - i. Is there an association – Independence

**We either Reject the  $H_0$  or Fail to Reject  $H_0$ . NEVER SAY WE ACCEPT THE  $H_0$ .**

**If the question states:** calculate/construction a \_\_\_\_% confidence interval...**Do a confidence interval**

## Confidence Intervals

- **Steps to constructing an Confidence Interval:**

1. Name the test
  - 1 Proportion Z Interval or 2 Proportion Z Interval
  - 1 Sample T Interval or 2 Sample T Interval
2. Conditions
  - Proportion
    - Randomness;
    - Normality
      - The scenario says its normal or
      -
  - Means
    - Randomness;
    - Normality
      - The scenario says its normal or
      -
3. Calculations/Intervals – C.I. = statistic  $\pm$  critical value (standard deviation of the statistic)
  - List given values and the interval from the calculator
4. Interpretation– Write your statement in context.
  - We are #% confident that the true population [mean or proportion] [context] **would** between [a] and [b].

- **Point Estimate** – uses a single statistic based on sample data, this is the simplest approach. (
- **Confidence Intervals** – used to estimate the unknown population parameter.
- **Margin of Error** – the smaller the margin of error, the more precise our estimate

## What makes the margin of error smaller

- make critical value smaller (lower confidence level).
- get a sample with a smaller standard deviation.
- make sample size (n) larger.

**To assess Normality** – Use graphs –boxplots, histograms, dotplots, or normal probability plot.

# Things to Remember

## Hypothesis Tests

- **Steps to running a hypothesis test:**

1. Name the test
  - 1 Proportion Z Test or 2 Proportion Z Test
  - 1 Sample T Test or 2 Sample T Test or Paired T-Test
2. Define Parameters and Write hypothesis
  - $p$  = true proportion context of the problem
  - $\mu$  = true mean context of the problem.
  - $H_0 =$
  - $H_a$
3. Conditions
  - Proportion
    - Randomness;
    - Normality
      - The scenario says its normal or
      -
  - Means
    - Randomness;
    - Normality
      - The scenario says its normal or
      -
4. Calculations
  - List given values and, z or t test statistic & p-value
5. Conclusion – Write your statement in context.
  - Since the p-value is  $< \alpha$ , we reject the  $H_0$ . There is sufficient evidence to suggest that [context of  $H_a$ /problem].
  - Since the p-value is  $> \alpha$ , we fail to reject the  $H_0$ . There is not sufficient evidence to suggest that [context of  $H_a$ /problem].

- **P-Value**

1. How to interpret: assuming the null is true, the probability of obtaining the observed result or more extreme

- **Level of Significance** –  $\alpha$  is the amount of evidence necessary before rejecting or failing to reject the null hypothesis.

## Type I and II Errors and Power

- **Type I Error** – is when we reject  $H_0$  when  $H_0$  is actually true.
- **Type II Error** – is when we fail to reject  $H_0$ , and  $H_a$  is actually true.
- **The Power of a Test** – is the probability that the test will reject the null hypothesis when the null hypothesis is false assuming the null is true. Power =  $1 - \beta$

# Things to Remember

	Type I error	Type II error	Power
<b>If you increase <math>\alpha</math></b>	Increases	Decreases	Increases
<b>If you increase n</b>	Increases	Decreases	Increases

## Chi-Square

- **$\chi^2$  Test** – is used to test counts of categorical data.
- **Goodness of Fit** – is for univariate categorical data from a single sample. Does the observed count “fit” what we expect. Must use list to perform,  $df = \text{number of the categories} - 1$
- **Independence** – bivariate categorical data. Are the two variables independent or is there an association? Use matrices to calculate
- **Homogeneity** -single categorical variable from 2 (or more) samples. Are distributions homogeneous or is there a difference? Use matrices to calculate

## Steps to running a chi-square test:

1. Name the test
  - Chi square goodness of fit
  - Chi square independence
  - Chi square homogeneity
2. Write hypothesis

Independence/Homogeneity

  - $H_0$ : There is no **association/difference** between contexts of the problem.
  - $H_a$ : There is **association/difference** between context of the problem.

Goodness of Fit

  - $H_0$ : The contexts of the problem is not as described or not equally likely
  - $H_a$ : The contexts of the problem is as described or equally likely
3. Conditions
  - Randomness;
  - Expected Counts
    - All expected counts must be greater than or equal to 5.
4. Calculations

$\chi^2$  value and degree of freedom.

  - Goodness of fit ( $df = \text{categories} - 1$ )
  - Independence/Homogeneity ( $df = (r-1)(c-1)$ )
5. Conclusion – Write your statement in context.
  - Since the p-value is  $< \alpha$ , we reject the  $H_0$ . There is sufficient evidence to suggest that [context of  $H_a$ /problem].
  - Since the p-value is  $> \alpha$ , we fail to reject the  $H_0$ . There is not sufficient evidence to suggest that [context of  $H_a$ /problem].

# Calculator Key Presses



Function	Key Press
<p><b><u>When to use data vs when to use stats</u></b></p> <ul style="list-style-type: none"> <li>• When given actual data, choose <b>Data</b></li> <li>• When given summary statistics (mean, standard deviation, proportion, percentage), choose <b>STATS</b></li> </ul> <p><b><u>When to use a maxtrix</u></b></p> <ul style="list-style-type: none"> <li>• For Chi-squared Independence/Homogeneity.</li> </ul> <p><b><u>Tests/Intervals</u></b></p> <ol style="list-style-type: none"> <li><b>Z Test</b> <ul style="list-style-type: none"> <li>• Use for one sample means problems where the population standard deviation is known <b>Hypothesis Test</b></li> </ul> </li> <li><b>T Test</b> <ul style="list-style-type: none"> <li>• Use for one sample means problems or for paired test problems</li> <li>• Use when population standard deviation is unknown <b>Hypothesis Test</b></li> </ul> </li> <li><b>2 Samp Z Test</b> <ul style="list-style-type: none"> <li>• Use for two sample means problems where the population standard deviation is known <b>Hypothesis Test</b></li> </ul> </li> <li><b>2 Samp T Test</b> <ul style="list-style-type: none"> <li>• Use for two sample means problems when population standard deviation is unknown <b>Hypothesis Test</b></li> </ul> </li> <li><b>1 Prop Z Test</b> <ul style="list-style-type: none"> <li>• Use for one sample proportion problems <b>Hypothesis Test</b></li> </ul> </li> <li><b>2 Prop Z Test</b> <ul style="list-style-type: none"> <li>• Use for two sample proportion problems <b>Hypothesis Test</b></li> </ul> </li> <li><b>Z Interval</b> <ul style="list-style-type: none"> <li>• <b>Never use</b></li> </ul> </li> <li><b>T Interval</b> <ul style="list-style-type: none"> <li>• Use for one sample means <b>confidence interval</b> problems</li> </ul> </li> <li><b>2 Samp Z Int</b> <ul style="list-style-type: none"> <li>• <b>Never Use</b></li> </ul> </li> <li><b>2 Samp T Int</b> <ul style="list-style-type: none"> <li>• Use for two sample means <b>confidence interval</b> problems</li> </ul> </li> <li><b>A. 1 Prop Z Int</b> <ul style="list-style-type: none"> <li>• Use for one sample proportion <b>confidence interval</b> problems</li> </ul> </li> <li><b>B. 2 Prop Z Int</b> <ul style="list-style-type: none"> <li>• Use for two sample proportion <b>confidence interval</b> problems</li> </ul> </li> <li><b>C. <math>\chi^2</math> – Test</b> <ul style="list-style-type: none"> <li>• Use for chi-squared test for independence or homogeneity problems</li> <li>• Table of categorical data</li> </ul> </li> <li><b>D. <math>\chi^2</math> – GOF Test</b> <ul style="list-style-type: none"> <li>• Use for chi-squared goodness of fit problems</li> <li>• Single list of categorical data</li> </ul> </li> </ol>	<p><b>Key Press</b></p> <p><b>Z-Test</b> Inpt: <b>Data</b> Stats</p> <ul style="list-style-type: none"> <li>• Stat</li> <li>• Test</li> <li>• Choose the appropriate test</li> </ul>

<b>Calculator Name for Test</b>	<b>The Appropriate Statistical Inference Test</b>
• Z-Test	
• T- Test	
• 2 – SampZTest	
• 2 – SampTTest	
• 1 – PropZTest	
• 2 – PropZTest	
• ZInterval	
• TInterval	
• 2 – SampZInt	
• 2 – SampTInt	
• 1 – PropZInt	
• 2 – PropZInt	
• $\chi^2$ Test	
• $\chi^2$ GOF - Test	
• 2 – SampleF Test	
• Lin RegTTest	
• LinRegTInt	
• ANOVA	





# Inference Review Problems

1. Researchers want to determine whether drivers are significantly more distracted while driving when using a cell phone than when talking to a passenger in the car. In a study involving 48 people, 24 people were randomly assigned to drive in a driving simulator while using a cell phone. The remaining 24 were assigned to drive in a driving simulator while talking to a passenger in the simulator. Part of the driving simulation for both groups involved asking drivers to exit the freeway at a particular exit. In the study, 7 of the 24 cell phone users missed the exit, while 2 of the 24 talking to a passenger missed the exit.

(a) Would this study be classified as an experiment or an observational study?

(b) Define the parameter(s).

(c) State the null and alternate hypotheses of interest to the researchers.

(d) One test of significance that you might consider using to answer the researchers' question is a two-sample  $z$ -test. State the conditions required for this test to be appropriate. Then comment on whether each condition is met.

(e) Using an advanced statistical method for small samples to test the hypotheses in part (b), the researchers report a  $p$ -value of 0.0683. State what conclusion should be made based on this  $p$ -value.

2. Suppose that a random sample of 50 bottles of a particular brand of cough medicine is selected and the alcohol content of each bottle is determined. Suppose the sample of 50 bottles has an average of 8.2 grams of alcohol content with a standard deviation of 1.5 grams.
- (a) Find the 95% confidence interval for the mean alcohol content of the cough medicine.

(b) Explain in words what the 95% confidence interval means.

(c) Would the 90% confidence interval be narrower or wider? Explain why.

**(d)** The manufacturer claims that the alcohol content is more than 8.0 grams per bottle. Is there statistical evidence that validate the manufacturer's claim that alcohol content is more than 8.0 grams per bottle/

3. Retailers report that the use of cents-off coupons is increasing. The Scripps Howard News Service reported that proportion of all households that use coupons as 0.77. Suppose that this estimate was based on a random sample of 800 households.
- (a) Construct the 99% confidence interval for the true population proportion. Show all work.

- (b) The manager of the retail store in reporting to his superiors claims that the true proportion of customers that use coupons is 80%. Test the manager's claim.

4. A random sample of 400 married couples was selected from a large population of married couples.
- Heights of married men are approximately normally distributed with mean 70 inches and standard deviation 3 inches.
  - Heights of married women are approximately normally distributed with mean 65 inches and standard deviation 2.5 inches.
  - There were 20 couples in which the wife was taller than her husband, and there were 380 couples in which the wife was shorter than her husband.
- (a) Find a 95 percent confidence interval for the proportion of married couples in the population for which the wife is taller than her husband. Interpret your interval in the context of this question.



7. After verifying that the conditions for inference were satisfied, the administrator performed a chi-square test of the following hypotheses.
- $H_0$ : There is no association between residential status and level of participation in extracurricular activities among the students at the university.
- $H_a$ : There is an association between residential status and level of participation in extracurricular activities among the students at the university.

The test resulted in a p-value of 0.23. Based on the p-value, what conclusion should the administrator make?

8. The manager of a local fast-food restaurant is concerned about customers who ask for a water cup when placing an order but fill the cup with a soft drink from the beverage fountain instead of filling the cup with water. The manager selected a random sample of 80 customers who asked for a water cup when placing an order and found that 23 of those customers filled the cup with a soft drink from the beverage fountain.
- Construct and interpret a 95 percent confidence interval for the proportion of all customers who, having asked for a water cup when placing an order, will fill the cup with a soft drink from the beverage fountain.

9. A 2008 study was undertaken to compare people who were overweight and their exercise habits. It was theorized that overweight people who kept track of their daily food intake by keeping records lost more weight than those who didn't keep records. Here are the statistics from the study:

	Number of participants	Pre-study		Post-study	
		Avg. Weight	St. Dev.	Avg. Weight	St. Dev.
People who did not keep records	75	235	23	218	20
People who kept records	84	241	24	219	19

What conclusions can be made from the study at 5% significance level?

10. High cholesterol levels in people can be reduced by exercise, diet, and medication. Twenty middle-aged males with cholesterol readings between 220 and 240 milligrams per deciliter (mg/dL) of blood were randomly selected from the population of such male patients at a large local hospital. Ten of the 20 males were randomly assigned to group A, advised on appropriate exercise and diet, and also received a placebo. The other 10 males were assigned to group B, received the same advice on appropriate exercise and diet, but received a drug intended to reduce cholesterol instead of a placebo. After three months, posttreatment cholesterol readings were taken for all 20 males and compared to pretreatment cholesterol readings. The tables below give the reduction in cholesterol level (pretreatment reading minus posttreatment reading) for each male in the study.

Group A (placebo)

Reduction (in mg/dL)	2	19	8	4	12	8	17	7	24	1
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Mean Reduction: 10.20    Standard Deviation of Reductions: 7.66

Group B (cholesterol drug)

Reduction (in mg/dL)	30	19	18	17	20	-4	23	10	9	22
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Mean Reduction: 16.40    Standard Deviation of Reductions: 9.40

Do the data provide convincing evidence, at the  $0.01 = \alpha$  level, that the cholesterol drug is effective in producing a reduction in mean cholesterol level beyond that produced by exercise and diet?

11. Party University tested its students' fitness. It tested strength and aerobic capacities of students and rated them as low, medium low, medium high, and high. On the questionnaire that students completed, it asked them about drinking habits on weekends. Following are the results of the study:

	Fitness Level			
	Low	Med Low	Med High	High
Never drink	238	173	148	77
1 drink only	209	177	125	81
More than 1 drink	294	201	154	58

What can be said about the relationship between fitness level and drinking habits on weekends (assuming students are honest about drinking habits)?

12. It is said that Internet surveys are unpredictable, but how about an Internet survey about using the Internet? For a portion of a day, Google kept track of computers logged into its site to see what type of computer and what Internet browser was used. The results are as follows.

	Explorer	Safari	Firefox	Other
PC	2,453	1,294	276	255
Mac	954	1,853	623	162

Find and interpret a 95% confidence interval for the difference in percentage between computers that use Explorer as their browser.