CALCULUS I

Practice Problems

Review

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Table of Contents

Preface ..... **ii**

Chapter 1 : Review **1**

Section 1-1 : Functions 3
Section 1-2 : Inverse Functions 6
Section 1-3 : Trig Functions 7
Section 1-4 : Solving Trig Equations 9
Section 1-5 : Solving Trig Equations with Calculators, Part I 10
Section 1-6 : Solving Trig Equations with Calculators, Part II 11
Section 1-7 : Exponential Functions 12
Section 1-8 : Logarithm Functions 13
Section 1-9 : Exponential and Logarithm Equations 15
Section 1-10 : Common Graphs 17

Preface

Here are a set of practice problems for the Calculus I notes. If you are viewing the pdf version of this document (as opposed to viewing it on the web) this document contains only the problems themselves and no solutions are included in this document. Solutions can be found in a couple of places on the site.

1. If you'd like a pdf document containing the solutions the download tab above contains links to pdf's containing the solutions for the full book, chapter and section. At this time, I do not offer pdf's for solutions to individual problems.

2. If you'd like to view the solutions on the web go to the problem set web page, click the solution link for any problem and it will take you to the solution to that problem.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Chapter 1 : Review

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Here is a list of all the sections for which practice problems have been written as well as a brief description of the material covered in the notes for that particular section.

Functions – In this section we will cover function notation/evaluation, determining the domain and range of a function and function composition.

Inverse Functions – In this section we will define an inverse function and the notation used for inverse functions. We will also discuss the process for finding an inverse function.

Trig Functions – In this section we will give a quick review of trig functions. We will cover the basic notation, relationship between the trig functions, the right triangle definition of the trig functions. We will also cover evaluation of trig functions as well as the unit circle (one of the most important ideas from a trig class!) and how it can be used to evaluate trig functions.

Solving Trig Equations – In this section we will discuss how to solve trig equations. The answers to the equations in this section will all be one of the "standard" angles that most students have memorized after a trig class. However, the process used here can be used for any answer regardless of it being one of the standard angles or not.

Solving Trig Equations with Calculators, Part I – In this section we will discuss solving trig equations when the answer will (generally) require the use of a calculator (i.e. they aren't one of the standard angles). Note however, the process used here is identical to that for when the answer is one of the standard angles. The only difference is that the answers in here can be a little messy due to the need of a calculator. Included is a brief discussion of inverse trig functions.

Solving Trig Equations with Calculators, Part II – In this section we will continue our discussion of solving trig equations when a calculator is needed to get the answer. The equations in this section tend

to be a little trickier than the "normal" trig equation and are not always covered in a trig class.

Exponential Functions – In this section we will discuss exponential functions. We will cover the basic definition of an exponential function, the natural exponential function, i.e. $\mathbf{e}^x$, as well as the properties and graphs of exponential functions.

Logarithm Functions – In this section we will discuss logarithm functions, evaluation of logarithms and their properties. We will discuss many of the basic manipulations of logarithms that commonly occur in Calculus (and higher) classes. Included is a discussion of the natural (\(\ln(x)\)) and common logarithm (\(\log(x)\)) as well as the change of base formula.

Exponential and Logarithm Equations – In this section we will discuss various methods for solving equations that involve exponential functions or logarithm functions.

Common Graphs – In this section we will do a very quick review of many of the most common functions and their graphs that typically show up in a Calculus class.

Section 1-1 : Functions

For problems 1 – 4 the given functions perform the indicated function evaluations.

1. $f(x)=3-5x-2x^2$
   (a) $f(4)$ (b) $f(0)$ (c) $f(-3)$
   (d) $f(6-t)$ (e) $f(7-4x)$ (f) $f(x+h)$

2. $g(t)=\dfrac{t}{2t+6}$
   (a) $g(0)$ (b) $g(-3)$ (c) $g(10)$
   (d) $g(x^2)$ (e) $g(t+h)$ (f) $g(t^2-3t+1)$

3. $h(z)=\sqrt{1-z^2}$
   (a) $h(0)$ (b) $h(-\frac{1}{2})$ (c) $h(\frac{1}{2})$
   (d) $h(9z)$ (e) $h(z^2-2z)$ (f) $h(z+k)$

4. $R(x)=\sqrt{3+x}-\dfrac{4}{x+1}$
   (a) $R(0)$ (b) $R(6)$ (c) $R(-9)$
   (d) $R(x+1)$ (e) $R(x^4-3)$ (f) $R(\frac{1}{x}-1)$

The **difference quotient** of a function $f(x)$ is defined to be,

$$\frac{f(x+h)-f(x)}{h}$$

For problems 5 – 9 compute the difference quotient of the given function.

5. $f(x)=4x-9$

6. $g(x)=6-x^2$

7. $f(t)=2t^2-3t+9$

8. $y(z)=\dfrac{1}{z+2}$

9. $A(t)=\dfrac{2t}{3-t}$

For problems 10 – 17 determine all the roots of the given function.

10. $f(x)=x^5-4x^4-32x^3$

11. $R(y)=12y^2+11y-5$

12. $h(t)=18-3t-2t^2$

13. $g(x)=x^3+7x^2-x$

14. $W(x)=x^4+6x^2-27$

15. $f(t)=t^{\frac{5}{3}}-7t^{\frac{4}{3}}-8t$

16. $h(z)=\dfrac{z}{z-5}-\dfrac{4}{z-8}$

17. $g(w)=\dfrac{2w}{w+1}+\dfrac{w-4}{2w-3}$

For problems 18 – 22 find the domain and range of the given function.

18. $Y(t)=3t^2-2t+1$

19. $g(z)=-z^2-4z+7$

20. $f(z)=2+\sqrt{z^2+1}$

21. $h(y)=-3\sqrt{14+3y}$

22. $M(x)=5-|x+8|$

For problems 23 – 32 find the domain of the given function.

23. $f(w)=\dfrac{w^3-3w+1}{12w-7}$

24. $R(z)=\dfrac{5}{z^3+10z^2+9z}$

25. $g(t)=\dfrac{6t-t^3}{7-t-4t^2}$

26. $g(x)=\sqrt{25-x^2}$

27. $h(x)=\sqrt{x^4-x^3-20x^2}$

28. $P(t)=\dfrac{5t+1}{\sqrt{t^3-t^2-8t}}$

29. $f(z)=\sqrt{z-1}+\sqrt{z+6}$

30. $h(y)=\sqrt{2y+9}-\dfrac{1}{\sqrt{2-y}}$

31. $A(x)=\dfrac{4}{x-9}-\sqrt{x^2-36}$

32. $Q(y)=\sqrt{y^2+1}-\sqrt[3]{1-y}$

For problems 33 – 36 compute $(f\circ g)(x)$ and $(g\circ f)(x)$ for each of the given pair of functions.

33. $f(x)=4x-1$, $g(x)=\sqrt{6+7x}$

34. $f(x)=5x+2$, $g(x)=x^2-14x$

35. $f(x)=x^2-2x+1$, $g(x)=8-3x^2$

36. $f(x)=x^2+3$, $g(x)=\sqrt{5+x^2}$

Section 1-2 : Inverse Functions

For each of the following functions find the inverse of the function. Verify your inverse by computing one or both of the composition as discussed in this section.

1. $f\left(x\right)=6x+15$

2. $h\left(x\right)=3-29x$

3. $R\left(x\right)=x^3+6$

4. $g\left(x\right)=4\left(x-3\right)^5+21$

5. $W\left(x\right)=\sqrt[5]{9-11x}$

6. $f\left(x\right)=\sqrt[7]{5x+8}$

7. $h\left(x\right)=\dfrac{1+9x}{4-x}$

8. $f\left(x\right)=\dfrac{6-10x}{8x+7}$

Section 1-3 : Trig Functions

Determine the exact value of each of the following without using a calculator.

Note that the point of these problems is not really to learn how to find the value of trig functions but instead to get you comfortable with the unit circle since that is a very important skill that will be needed in solving trig equations.

1. $\cos\left(\frac{5\pi}{6}\right)$

2. $\sin\left(-\frac{4\pi}{3}\right)$

3. $\sin\left(\frac{7\pi}{4}\right)$

4. $\cos\left(-\frac{2\pi}{3}\right)$

5. $\tan\left(\frac{3\pi}{4}\right)$

6. $\sec\left(-\frac{11\pi}{6}\right)$

7. $\cos\left(\frac{8\pi}{3}\right)$

8. $\tan\left(-\frac{\pi}{3}\right)$

9. $\tan\left(\frac{15\pi}{4}\right)$

10. $\sin\left(-\frac{11\pi}{3}\right)$

11. $\sec\left(\frac{29\pi}{4}\right)$

Section 1-4 : Solving Trig Equations

Without using a calculator find the solution(s) to the following equations. If an interval is given find only those solutions that are in the interval. If no interval is given find all solutions to the equation.

1. $4\sin\left(3t\right)=2$

2. $4\sin\left(3t\right)=2$ in $\left[0,\frac{4\pi}{3}\right]$

3. $2\cos\left(\frac{x}{3}\right)+\sqrt{2}=0$

4. $2\cos\left(\frac{x}{3}\right)+\sqrt{2}=0$ in $\left[-7\pi,7\pi\right]$

5. $4\cos\left(6z\right)=\sqrt{12}$ in $\left[0,\frac{\pi}{2}\right]$

6. $2\sin\left(\frac{3y}{2}\right)+\sqrt{3}=0$ in $\left[-\frac{7\pi}{3},0\right]$

7. $8\tan\left(2x\right)-5=3$ in $\left[-\frac{\pi}{2},\frac{3\pi}{2}\right]$

8. $16=-9\sin\left(7x\right)-4$ in $\left[-2\pi,\frac{9\pi}{4}\right]$

9. $\sqrt{3}\tan\left(\frac{t}{4}\right)+5=4$ in $\left[0,4\pi\right]$

10. $\sqrt{3}\csc\left(9z\right)-7=-5$ in $\left[-\frac{\pi}{3},\frac{4\pi}{9}\right]$

11. $1-14\cos\left(\frac{2x}{5}\right)=-6$ in $\left[5\pi,\frac{40\pi}{3}\right]$

12. $15=17+4\cos\left(\frac{y}{7}\right)$ in $\left[10\pi,15\pi\right]$

Section 1-5 : Solving Trig Equations with Calculators, Part I

Find the solution(s) to the following equations. If an interval is given find only those solutions that are in the interval. If no interval is given find all solutions to the equation. These will require the use of a calculator so use at least 4 decimal places in your work.

1. $7\cos\left(4x\right)+11=10$

2. $6+5\cos\left(\frac{x}{3}\right)=10$ in $\left[0,38\right]$

3. $3=6-11\sin\left(\frac{t}{8}\right)$

4. $4\sin\left(6z\right)+\frac{13}{10}=-\frac{3}{10}$ in $\left[0,2\right]$

5. $9\cos\left(\frac{4z}{9}\right)+21\sin\left(\frac{4z}{9}\right)=0$ in $\left[-10,10\right]$

6. $3\tan\left(\frac{w}{4}\right)-1=11-2\tan\left(\frac{w}{4}\right)$ in $\left[-50,0\right]$

7. $17-3\sec\left(\frac{z}{2}\right)=2$ in $\left[20,45\right]$

8. $12\sin\left(7y\right)+11=3+4\sin\left(7y\right)$ in $\left[-2,-\frac{1}{2}\right]$

9. $5-14\tan\left(8x\right)=30$ in $\left[-1,1\right]$

10. $0=18+2\csc\left(\frac{t}{3}\right)$ in $\left[0,5\right]$

11. $\frac{1}{2}\cos\left(\frac{x}{8}\right)+\frac{1}{4}=\frac{2}{3}$ in $\left[0,100\right]$

12. $\frac{4}{3}=1+3\sec\left(2t\right)$ in $\left[-4,6\right]$

Section 1-6 : Solving Trig Equations with Calculators, Part II

Find all the solution(s) to the following equations. These will require the use of a calculator so use at least 4 decimal places in your work.

1. $3-14\sin\left(12t+7\right)=13$

2. $3\sec\left(4-9z\right)-24=0$

3. $4\sin\left(x+2\right)-15\sin\left(x+2\right)\tan\left(4x\right)=0$

4. $3\cos\left(\frac{3y}{7}\right)\sin\left(\frac{y}{2}\right)+14\cos\left(\frac{3y}{7}\right)=0$

5. $7\cos^2\left(3x\right)-\cos\left(3x\right)=0$

6. $\tan^2\left(\frac{w}{4}\right)=\tan\left(\frac{w}{4}\right)+12$

7. $4\csc^2\left(1-t\right)+6=25\csc\left(1-t\right)$

8. $4y\sec\left(7y\right)=-21y$

9. $10x^2\sin\left(3x+2\right)=7x\sin\left(3x+2\right)$

10. $\left(2t-3\right)\tan\left(\frac{6t}{11}\right)=15-10t$

Section 1-7 : Exponential Functions

Sketch the graphs of each of the following functions.

1. $f\left(x\right)=3^{1+2x}$

2. $h\left(x\right)=2^{3-\frac{x}{4}}-7$

3. $h\left(t\right)=8+3\mathbf{e}^{2t-4}$

4. $g\left(z\right)=10-\frac{1}{4}\mathbf{e}^{-2-3z}$

Section 1-8 : Logarithm Functions

Without using a calculator determine the exact value of each of the following.

1. $\log_3 81$

2. $\log_5 125$

3. $\log_2 \frac{1}{8}$

4. $\log_{\frac{1}{4}} 16$

5. $\ln \mathbf{e}^4$

6. $\log \frac{1}{100}$

Write each of the following in terms of simpler logarithms

7. $\log\left(3x^4 y^{-7}\right)$

8. $\ln\left(x\sqrt{y^2+z^2}\right)$

9. $\log_4\left(\frac{x-4}{y^2\sqrt[5]{z}}\right)$

Combine each of the following into a single logarithm with a coefficient of one.

10. $2\log_4 x + 5\log_4 y - \frac{1}{2}\log_4 z$

11. $3\ln\left(t+5\right) - 4\ln t - 2\ln\left(s-1\right)$

12. $\frac{1}{3}\log a - 6\log b + 2$

Use the change of base formula and a calculator to find the value of each of the following.

13. $\log_{12} 35$

14. $\log_{\frac{2}{3}} 53$

Section 1-9 : Exponential and Logarithm Equations

For problems 1 – 12 find all the solutions to the given equation. If there is no solution to the equation clearly explain why.

1. $12-4\mathbf{e}^{7+3x}=7$

2. $1=10-3\mathbf{e}^{z^2-2z}$

3. $2t-t\mathbf{e}^{6t-1}=0$

4. $4x+1=\left(12x+3\right)\mathbf{e}^{x^2-2}$

5. $2\mathbf{e}^{3y+8}-11\mathbf{e}^{5-10y}=0$

6. $14\mathbf{e}^{6-x}+\mathbf{e}^{12x-7}=0$

7. $1-8\ln\left(\frac{2x-1}{7}\right)=14$

8. $\ln\left(y-1\right)=1+\ln\left(3y+2\right)$

9. $\log\left(w\right)+\log\left(w-21\right)=2$

10. $2\log\left(z\right)-\log\left(7z-1\right)=0$

11. $16=17^{t-2}+11$

12. $2^{3-8w}-7=11$

Compound Interest. If we put *P* dollars into an account that earns interest at a rate of *r* (written as a decimal as opposed to the standard percent) for *t* years then,

a. if interest is compounded *m* times per year we will have,

$$A=P\left(1+\frac{r}{m}\right)^{tm}$$

dollars after *t* years.

b. if interest is compounded continuously we will have,

$$A=P\mathbf{e}^{rt}$$

dollars after *t* years.

13. We have $10,000 to invest for 44 months. How much money will we have if we put the money into an account that has an annual interest rate of 5.5% and interest is compounded
(a) quarterly (b) monthly (c) continuously

14. We are starting with $5000 and we're going to put it into an account that earns an annual interest rate of 12%. How long should we leave the money in the account in order to double our money if interest is compounded
(a) quarterly (b) monthly (c) continuously

Exponential Growth/Decay. Many quantities in the world can be modeled (at least for a short time) by the exponential growth/decay equation.

$$Q = Q_0 \mathbf{e}^{kt}$$

If *k* is positive we will get exponential growth and if *k* is negative we will get exponential decay.

15. A population of bacteria initially has 250 present and in 5 days there will be 1600 bacteria present.
(a) Determine the exponential growth equation for this population.
(b) How long will it take for the population to grow from its initial population of 250 to a population of 2000?

16. We initially have 100 grams of a radioactive element and in 1250 years there will be 80 grams left.
(a) Determine the exponential decay equation for this element.
(b) How long will it take for half of the element to decay?
(c) How long will it take until there is only 1 gram of the element left?

Section 1-10 : Common Graphs

Without using a graphing calculator sketch the graph of each of the following.

1. $y = \frac{4}{3}x - 2$

2. $f(x) = |x - 3|$

3. $g(x) = \sin(x) + 6$

4. $f(x) = \ln(x) - 5$

5. $h(x) = \cos\left(x + \frac{\pi}{2}\right)$

6. $h(x) = (x-3)^2 + 4$

7. $W(x) = \mathbf{e}^{x+2} - 3$

8. $f(y) = (y-1)^2 + 2$

9. $R(x) = -\sqrt{x}$

10. $g(x) = \sqrt{-x}$

11. $h(x) = 2x^2 - 3x + 4$

12. $f(y) = -4y^2 + 8y + 3$

13. $(x+1)^2 + (y-5)^2 = 9$

14. $x^2 - 4x + y^2 - 6y - 87 = 0$

15. $25(x+2)^2 + \frac{y^2}{4} = 1$

16. $x^2 + \frac{(y-6)^2}{9} = 1$

17. $\frac{x^2}{36} - \frac{y^2}{49} = 1$

18. $(y+2)^2 - \frac{(x+4)^2}{16} = 1$

