

CALCULUS I
Practice Problems
Limits

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Preface

Here are a set of practice problems for the Calculus I notes. If you are viewing the pdf version of this document (as opposed to viewing it on the web) this document contains only the problems themselves and no solutions are included in this document. Solutions can be found in a couple of places on the site.

1. If you'd like a pdf document containing the solutions the download tab above contains links to pdf's containing the solutions for the full book, chapter and section. At this time, I do not offer pdf's for solutions to individual problems.
2. If you'd like to view the solutions on the web go to the problem set web page, click the solution link for any problem and it will take you to the solution to that problem.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Chapter 2 : Limits

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Here is a list of all the sections for which practice problems have been written as well as a brief description of the material covered in the notes for that particular section.

Tangent Lines and Rates of Change – In this section we will introduce two problems that we will see time and again in this course : Rate of Change of a function and Tangent Lines to functions. Both of these problems will be used to introduce the concept of limits, although we won't formally give the definition or notation until the next section.

The Limit – In this section we will introduce the notation of the limit. We will also take a conceptual look at limits and try to get a grasp on just what they are and what they can tell us. We will be estimating the value of limits in this section to help us understand what they tell us. We will actually start computing limits in a couple of sections.

One-Sided Limits – In this section we will introduce the concept of one-sided limits. We will discuss the differences between one-sided limits and limits as well as how they are related to each other.

Limit Properties – In this section we will discuss the properties of limits that we'll need to use in computing limits (as opposed to estimating them as we've done to this point). We will also compute a couple of basic limits in this section.

Computing Limits – In this section we will look at several types of limits that require some work before we can use the limit properties to compute them. We will also look at computing limits of piecewise functions and use of the Squeeze Theorem to compute some limits.

Infinite Limits – In this section we will look at limits that have a value of infinity or negative infinity. We'll also take a brief look at vertical asymptotes.

Limits At Infinity, Part I – In this section we will start looking at limits at infinity, i.e. limits in which the variable gets very large in either the positive or negative sense. We will concentrate on polynomials and rational expressions in this section. We'll also take a brief look at horizontal asymptotes.

Limits At Infinity, Part II – In this section we will continue covering limits at infinity. We'll be looking at exponentials, logarithms and inverse tangents in this section.

Continuity – In this section we will introduce the concept of continuity and how it relates to limits. We will also see the Intermediate Value Theorem in this section and how it can be used to determine if functions have solutions in a given interval.

The Definition of the Limit – In this section we will give a precise definition of several of the limits covered in this section. We will work several basic examples illustrating how to use this precise definition to compute a limit. We'll also give a precise definition of continuity.

Section 2-1 : Tangent Lines and Rates of Change

1. For the function $f(x) = 3(x+2)^2$ and the point P given by $x = -3$ answer each of the following questions.

(a) For the points Q given by the following values of x compute (accurate to at least 8 decimal places) the slope, m_{PQ} , of the secant line through points P and Q .

(i) -3.5 (ii) -3.1 (iii) -3.01 (iv) -3.001 (v) -3.0001
(vi) -2.5 (vii) -2.9 (viii) -2.99 (ix) -2.999 (x) -2.9999

(b) Use the information from (a) to estimate the slope of the tangent line to $f(x)$ at $x = -3$ and write down the equation of the tangent line.

2. For the function $g(x) = \sqrt{4x+8}$ and the point P given by $x = 2$ answer each of the following questions.

(a) For the points Q given by the following values of x compute (accurate to at least 8 decimal places) the slope, m_{PQ} , of the secant line through points P and Q .

(i) 2.5 (ii) 2.1 (iii) 2.01 (iv) 2.001 (v) 2.0001
(vi) 1.5 (vii) 1.9 (viii) 1.99 (ix) 1.999 (x) 1.9999

(b) Use the information from (a) to estimate the slope of the tangent line to $g(x)$ at $x = 2$ and write down the equation of the tangent line.

3. For the function $W(x) = \ln(1+x^4)$ and the point P given by $x = 1$ answer each of the following questions.

(a) For the points Q given by the following values of x compute (accurate to at least 8 decimal places) the slope, m_{PQ} , of the secant line through points P and Q .

(i) 1.5 (ii) 1.1 (iii) 1.01 (iv) 1.001 (v) 1.0001
(vi) 0.5 (vii) 0.9 (viii) 0.99 (ix) 0.999 (x) 0.9999

(b) Use the information from (a) to estimate the slope of the tangent line to $W(x)$ at $x = 1$ and write down the equation of the tangent line.

4. The volume of air in a balloon is given by $V(t) = \frac{6}{4t+1}$ answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average rate of change of the volume of air in the balloon between $t = 0.25$ and the following values of t .

(i) 1 **(ii)** 0.5 **(iii)** 0.251 **(iv)** 0.2501 **(v)** 0.25001
(vi) 0 **(vii)** 0.1 **(viii)** 0.249 **(ix)** 0.2499 **(x)** 0.24999

(b) Use the information from **(a)** to estimate the instantaneous rate of change of the volume of air in the balloon at $t = 0.25$.

5. The population (in hundreds) of fish in a pond is given by $P(t) = 2t + \sin(2t - 10)$ answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average rate of change of the population of fish between $t = 5$ and the following values of t . Make sure your calculator is set to radians for the computations.

(i) 5.5 **(ii)** 5.1 **(iii)** 5.01 **(iv)** 5.001 **(v)** 5.0001
(vi) 4.5 **(vii)** 4.9 **(viii)** 4.99 **(ix)** 4.999 **(x)** 4.9999

(b) Use the information from **(a)** to estimate the instantaneous rate of change of the population of the fish at $t = 5$.

6. The position of an object is given by $s(t) = \cos^2\left(\frac{3t-6}{2}\right)$ answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average velocity of the object between $t = 2$ and the following values of t . Make sure your calculator is set to radians for the computations.

(i) 2.5 **(ii)** 2.1 **(iii)** 2.01 **(iv)** 2.001 **(v)** 2.0001
(vi) 1.5 **(vii)** 1.9 **(viii)** 1.99 **(ix)** 1.999 **(x)** 1.9999

(b) Use the information from **(a)** to estimate the instantaneous velocity of the object at $t = 2$ and determine if the object is moving to the right (*i.e.* the instantaneous velocity is positive), moving to the left (*i.e.* the instantaneous velocity is negative), or not moving (*i.e.* the instantaneous velocity is zero).

7. The position of an object is given by $s(t) = (8-t)(t+6)^{\frac{3}{2}}$. Note that a negative position here simply means that the position is to the left of the “zero position” and is perfectly acceptable. Answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average velocity of the object between $t = 10$ and the following values of t .

(i) 10.5 **(ii)** 10.1 **(iii)** 10.01 **(iv)** 10.001 **(v)** 10.0001
(vi) 9.5 **(vii)** 9.9 **(viii)** 9.99 **(ix)** 9.999 **(x)** 9.9999

(b) Use the information from **(a)** to estimate the instantaneous velocity of the object at $t = 10$ and determine if the object is moving to the right (*i.e.* the instantaneous velocity is positive), moving to the left (*i.e.* the instantaneous velocity is negative), or not moving (*i.e.* the instantaneous velocity is zero).

Section 2-2 : The Limit

1. For the function $f(x) = \frac{8-x^3}{x^2-4}$ answer each of the following questions.

(a) Evaluate the function the following values of x compute (accurate to at least 8 decimal places).

- (i) 2.5 (ii) 2.1 (iii) 2.01 (iv) 2.001 (v) 2.0001
 (vi) 1.5 (vii) 1.9 (viii) 1.99 (ix) 1.999 (x) 1.9999

(b) Use the information from (a) to estimate the value of $\lim_{x \rightarrow 2} \frac{8-x^3}{x^2-4}$.

2. For the function $R(t) = \frac{2-\sqrt{t^2+3}}{t+1}$ answer each of the following questions.

(a) Evaluate the function the following values of t compute (accurate to at least 8 decimal places).

- (i) -0.5 (ii) -0.9 (iii) -0.99 (iv) -0.999 (v) -0.9999
 (vi) -1.5 (vii) -1.1 (viii) -1.01 (ix) -1.001 (x) -1.0001

(b) Use the information from (a) to estimate the value of $\lim_{t \rightarrow -1} \frac{2-\sqrt{t^2+3}}{t+1}$.

3. For the function $g(\theta) = \frac{\sin(7\theta)}{\theta}$ answer each of the following questions.

(a) Evaluate the function the following values of θ compute (accurate to at least 8 decimal places).
 Make sure your calculator is set to radians for the computations.

- (i) 0.5 (ii) 0.1 (iii) 0.01 (iv) 0.001 (v) 0.0001
 (vi) -0.5 (vii) -0.1 (viii) -0.01 (ix) -0.001 (x) -0.0001

(b) Use the information from (a) to estimate the value of $\lim_{\theta \rightarrow 0} \frac{\sin(7\theta)}{\theta}$.

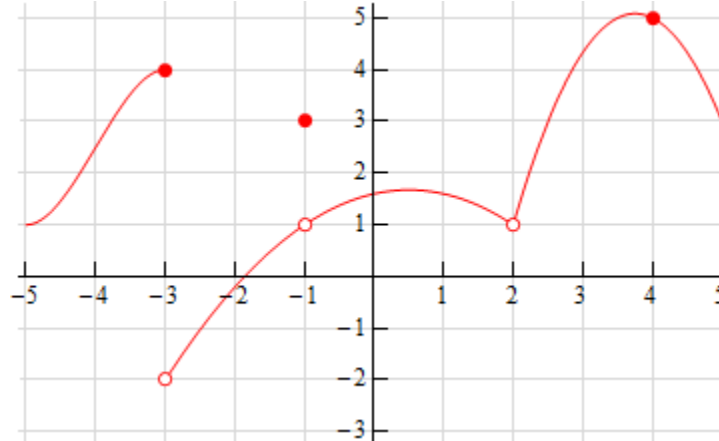
4. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$ and $\lim_{x \rightarrow a} f(x)$. If any of the quantities do not exist clearly explain why.

(a) $a = -3$

(b) $a = -1$

(c) $a = 2$

(d) $a = 4$



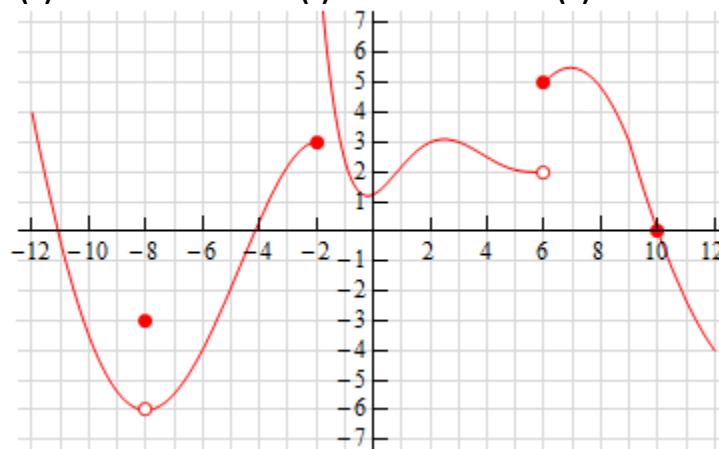
5. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$ and $\lim_{x \rightarrow a} f(x)$. If any of the quantities do not exist clearly explain why.

(a) $a = -8$

(b) $a = -2$

(c) $a = 6$

(d) $a = 10$



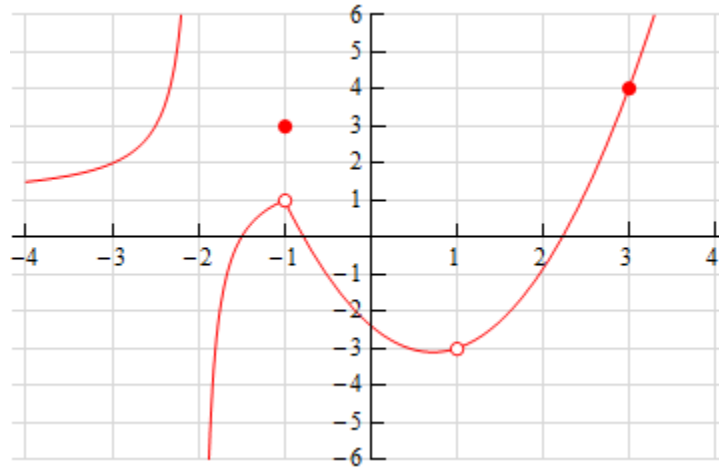
6. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$ and $\lim_{x \rightarrow a} f(x)$. If any of the quantities do not exist clearly explain why.

(a) $a = -2$

(b) $a = -1$

(c) $a = 1$

(d) $a = 3$



Section 2-3 : One-Sided Limits

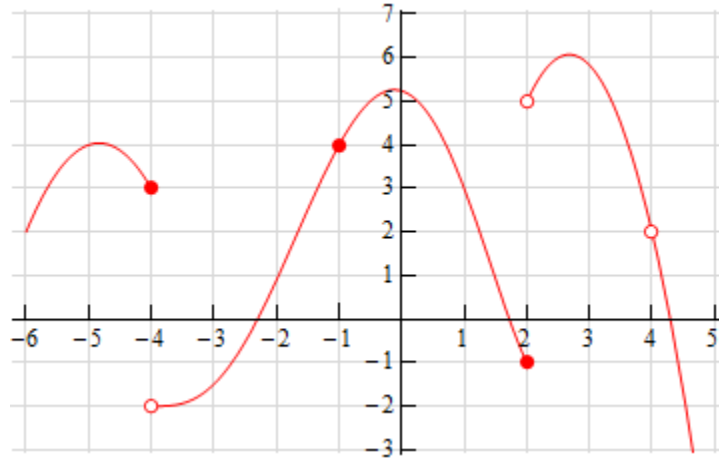
1. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$, $\lim_{x \rightarrow a^-} f(x)$, $\lim_{x \rightarrow a^+} f(x)$, and $\lim_{x \rightarrow a} f(x)$. If any of the quantities do not exist clearly explain why.

(a) $a = -4$

(b) $a = -1$

(c) $a = 2$

(d) $a = 4$



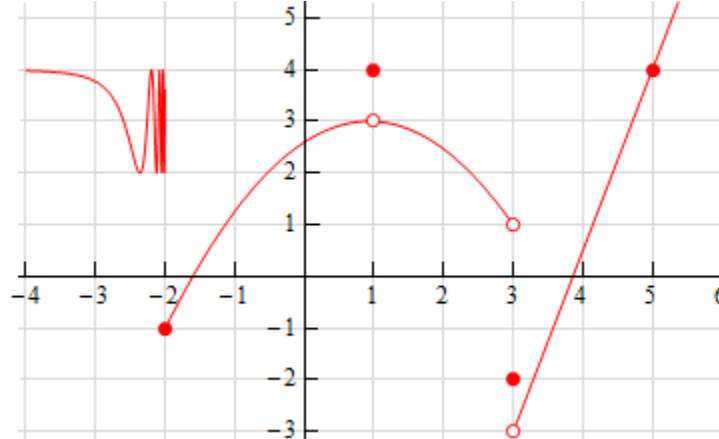
2. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$, $\lim_{x \rightarrow a^-} f(x)$, $\lim_{x \rightarrow a^+} f(x)$, and $\lim_{x \rightarrow a} f(x)$. If any of the quantities do not exist clearly explain why.

(a) $a = -2$

(b) $a = 1$

(c) $a = 3$

(d) $a = 5$



3. Sketch a graph of a function that satisfies each of the following conditions.

$$\lim_{x \rightarrow 2^-} f(x) = 1$$

$$\lim_{x \rightarrow 2^+} f(x) = -4$$

$$f(2) = 1$$

4. Sketch a graph of a function that satisfies each of the following conditions.

$$\lim_{x \rightarrow 3^-} f(x) = 0$$

$$\lim_{x \rightarrow 3^+} f(x) = 4$$

$f(3)$ does not exist

$$\lim_{x \rightarrow -1} f(x) = -3$$

$$f(-1) = 2$$

Section 2-4 : Limit Properties

1. Given $\lim_{x \rightarrow 8} f(x) = -9$, $\lim_{x \rightarrow 8} g(x) = 2$ and $\lim_{x \rightarrow 8} h(x) = 4$ use the limit properties given in this section to compute each of the following limits. If it is not possible to compute any of the limits clearly explain why not.

(a) $\lim_{x \rightarrow 8} [2f(x) - 12h(x)]$

(b) $\lim_{x \rightarrow 8} [3h(x) - 6]$

(c) $\lim_{x \rightarrow 8} [g(x)h(x) - f(x)]$

(d) $\lim_{x \rightarrow 8} [f(x) - g(x) + h(x)]$

2. Given $\lim_{x \rightarrow -4} f(x) = 1$, $\lim_{x \rightarrow -4} g(x) = 10$ and $\lim_{x \rightarrow -4} h(x) = -7$ use the limit properties given in this section to compute each of the following limits. If it is not possible to compute any of the limits clearly explain why not.

(a) $\lim_{x \rightarrow -4} \left[\frac{f(x)}{g(x)} - \frac{h(x)}{f(x)} \right]$

(b) $\lim_{x \rightarrow -4} [f(x)g(x)h(x)]$

(c) $\lim_{x \rightarrow -4} \left[\frac{1}{h(x)} + \frac{3 - f(x)}{g(x) + h(x)} \right]$

(d) $\lim_{x \rightarrow -4} \left[2h(x) - \frac{1}{h(x) + 7f(x)} \right]$

3. Given $\lim_{x \rightarrow 0} f(x) = 6$, $\lim_{x \rightarrow 0} g(x) = -4$ and $\lim_{x \rightarrow 0} h(x) = -1$ use the limit properties given in this section to compute each of the following limits. If it is not possible to compute any of the limits clearly explain why not.

(a) $\lim_{x \rightarrow 0} [f(x) + h(x)]^3$

(b) $\lim_{x \rightarrow 0} \sqrt{g(x)h(x)}$

(c) $\lim_{x \rightarrow 0} \sqrt[3]{11 + [g(x)]^2}$

(d) $\lim_{x \rightarrow 0} \sqrt{\frac{f(x)}{h(x) - g(x)}}$

For each of the following limits use the limit properties given in this section to compute the limit. At each step clearly indicate the property being used. If it is not possible to compute any of the limits clearly explain why not.

4. $\lim_{t \rightarrow -2} (14 - 6t + t^3)$

5. $\lim_{x \rightarrow 6} (3x^2 + 7x - 16)$

$$6. \lim_{w \rightarrow 3} \frac{w^2 - 8w}{4 - 7w}$$

$$7. \lim_{x \rightarrow -5} \frac{x + 7}{x^2 + 3x - 10}$$

$$8. \lim_{z \rightarrow 0} \sqrt{z^2 + 6}$$

$$9. \lim_{x \rightarrow 10} (4x + \sqrt[3]{x-2})$$

Section 2-5 : Computing Limits

For problems 1 – 9 evaluate the limit, if it exists.

1. $\lim_{x \rightarrow 2} (8 - 3x + 12x^2)$

2. $\lim_{t \rightarrow -3} \frac{6 + 4t}{t^2 + 1}$

3. $\lim_{x \rightarrow -5} \frac{x^2 - 25}{x^2 + 2x - 15}$

4. $\lim_{z \rightarrow 8} \frac{2z^2 - 17z + 8}{8 - z}$

5. $\lim_{y \rightarrow 7} \frac{y^2 - 4y - 21}{3y^2 - 17y - 28}$

6. $\lim_{h \rightarrow 0} \frac{(6+h)^2 - 36}{h}$

7. $\lim_{z \rightarrow 4} \frac{\sqrt{z} - 2}{z - 4}$

8. $\lim_{x \rightarrow -3} \frac{\sqrt{2x + 22} - 4}{x + 3}$

9. $\lim_{x \rightarrow 0} \frac{x}{3 - \sqrt{x + 9}}$

10. Given the function

$$f(x) = \begin{cases} 7 - 4x & x < 1 \\ x^2 + 2 & x \geq 1 \end{cases}$$

Evaluate the following limits, if they exist.

(a) $\lim_{x \rightarrow -6} f(x)$

(b) $\lim_{x \rightarrow 1} f(x)$

11. Given

$$h(z) = \begin{cases} 6z & z \leq -4 \\ 1-9z & z > -4 \end{cases}$$

Evaluate the following limits, if they exist.

(a) $\lim_{z \rightarrow 7} h(z)$

(b) $\lim_{z \rightarrow -4} h(z)$

For problems 12 & 13 evaluate the limit, if it exists.

12. $\lim_{x \rightarrow 5} (10 + |x - 5|)$

13. $\lim_{t \rightarrow -1} \frac{t+1}{|t+1|}$

14. Given that $7x \leq f(x) \leq 3x^2 + 2$ for all x determine the value of $\lim_{x \rightarrow 2} f(x)$.

15. Use the Squeeze Theorem to determine the value of $\lim_{x \rightarrow 0} x^4 \sin\left(\frac{\pi}{x}\right)$.

Section 2-6 : Infinite Limits

For problems 1 – 6 evaluate the indicated limits, if they exist.

1. For $f(x) = \frac{9}{(x-3)^5}$ evaluate,

(a) $\lim_{x \rightarrow 3^-} f(x)$

(b) $\lim_{x \rightarrow 3^+} f(x)$

(c) $\lim_{x \rightarrow 3} f(x)$

2. For $h(t) = \frac{2t}{6+t}$ evaluate,

(a) $\lim_{t \rightarrow -6^-} h(t)$

(b) $\lim_{t \rightarrow -6^+} h(t)$

(c) $\lim_{t \rightarrow -6} h(t)$

3. For $g(z) = \frac{z+3}{(z+1)^2}$ evaluate,

(a) $\lim_{z \rightarrow -1^-} g(z)$

(b) $\lim_{z \rightarrow -1^+} g(z)$

(c) $\lim_{z \rightarrow -1} g(z)$

4. For $g(x) = \frac{x+7}{x^2-4}$ evaluate,

(a) $\lim_{x \rightarrow 2^-} g(x)$

(b) $\lim_{x \rightarrow 2^+} g(x)$

(c) $\lim_{x \rightarrow 2} g(x)$

5. For $h(x) = \ln(-x)$ evaluate,

(a) $\lim_{x \rightarrow 0^-} h(x)$

(b) $\lim_{x \rightarrow 0^+} h(x)$

(c) $\lim_{x \rightarrow 0} h(x)$

6. For $R(y) = \tan(y)$ evaluate,

(a) $\lim_{y \rightarrow \frac{3\pi}{2}^-} R(y)$

(b) $\lim_{y \rightarrow \frac{3\pi}{2}^+} R(y)$

(c) $\lim_{y \rightarrow \frac{3\pi}{2}} R(y)$

For problems 7 & 8 find all the vertical asymptotes of the given function.

7. $f(x) = \frac{7x}{(10-3x)^4}$

8. $g(x) = \frac{-8}{(x+5)(x-9)}$

Section 2-7 : Limits at Infinity, Part I

1. For $f(x) = 4x^7 - 18x^3 + 9$ evaluate each of the following limits.

(a) $\lim_{x \rightarrow -\infty} f(x)$

(b) $\lim_{x \rightarrow \infty} f(x)$

2. For $h(t) = \sqrt[3]{t} + 12t - 2t^2$ evaluate each of the following limits.

(a) $\lim_{t \rightarrow -\infty} h(t)$

(b) $\lim_{t \rightarrow \infty} h(t)$

For problems 3 – 10 answer each of the following questions.

(a) Evaluate $\lim_{x \rightarrow -\infty} f(x)$.

(b) Evaluate $\lim_{x \rightarrow \infty} f(x)$.

(c) Write down the equation(s) of any horizontal asymptotes for the function.

3. $f(x) = \frac{8 - 4x^2}{9x^2 + 5x}$

4. $f(x) = \frac{3x^7 - 4x^2 + 1}{5 - 10x^2}$

5. $f(x) = \frac{20x^4 - 7x^3}{2x + 9x^2 + 5x^4}$

6. $f(x) = \frac{x^3 - 2x + 11}{3 - 6x^5}$

7. $f(x) = \frac{x^6 - x^4 + x^2 - 1}{7x^6 + 4x^3 + 10}$

8. $f(x) = \frac{\sqrt{7 + 9x^2}}{1 - 2x}$

9. $f(x) = \frac{x + 8}{\sqrt{2x^2 + 3}}$

10. $f(x) = \frac{8 + x - 4x^2}{\sqrt{6 + x^2 + 7x^4}}$

Section 2-8 : Limits At Infinity, Part II

For problems 1 – 6 evaluate (a) $\lim_{x \rightarrow -\infty} f(x)$ and (b) $\lim_{x \rightarrow \infty} f(x)$.

1. $f(x) = e^{8+2x-x^3}$

2. $f(x) = e^{\frac{6x^2+x}{5+3x}}$

3. $f(x) = 2e^{6x} - e^{-7x} - 10e^{4x}$

4. $f(x) = 3e^{-x} - 8e^{-5x} - e^{10x}$

5. $f(x) = \frac{e^{-3x} - 2e^{8x}}{9e^{8x} - 7e^{-3x}}$

6. $f(x) = \frac{e^{-7x} - 2e^{3x} - e^x}{e^{-x} + 16e^{10x} + 2e^{-4x}}$

For problems 7 – 12 evaluate the given limit.

7. $\lim_{t \rightarrow -\infty} \ln(4 - 9t - t^3)$

8. $\lim_{z \rightarrow -\infty} \ln\left(\frac{3z^4 - 8}{2 + z^2}\right)$

9. $\lim_{x \rightarrow \infty} \ln\left(\frac{11 + 8x}{x^3 + 7x}\right)$

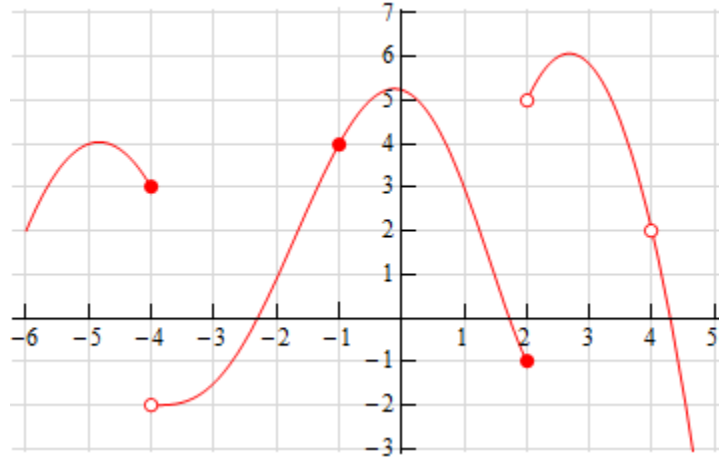
10. $\lim_{x \rightarrow -\infty} \tan^{-1}(7 - x + 3x^5)$

11. $\lim_{t \rightarrow \infty} \tan^{-1}\left(\frac{4 + 7t}{2 - t}\right)$

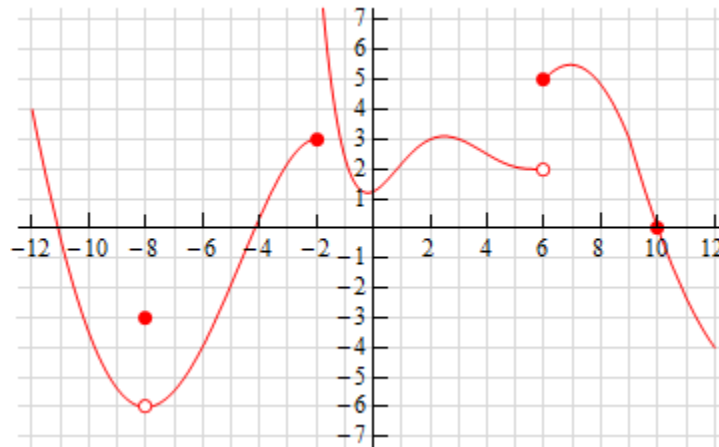
12. $\lim_{w \rightarrow \infty} \tan^{-1}\left(\frac{3w^2 - 9w^4}{4w - w^3}\right)$

Section 2-9 : Continuity

1. The graph of $f(x)$ is given below. Based on this graph determine where the function is discontinuous.



2. The graph of $f(x)$ is given below. Based on this graph determine where the function is discontinuous.



For problems 3 – 7 using only Properties 1 – 9 from the [Limit Properties](#) section, one-sided limit properties (if needed) and the definition of continuity determine if the given function is continuous or discontinuous at the indicated points.

$$3. f(x) = \frac{4x+5}{9-3x}$$

(a) $x = -1$, (b) $x = 0$, (c) $x = 3$

$$4. g(z) = \frac{6}{z^2 - 3z - 10}$$

(a) $z = -2$, (b) $z = 0$, (c) $z = 5$

$$5. g(x) = \begin{cases} 2x & x < 6 \\ x-1 & x \geq 6 \end{cases}$$

$$\text{(a) } x = 4, \text{ (b) } x = 6$$

$$6. h(t) = \begin{cases} t^2 & t < -2 \\ t+6 & t \geq -2 \end{cases}$$

$$\text{(a) } t = -2, \text{ (b) } t = 10$$

$$7. g(x) = \begin{cases} 1-3x & x < -6 \\ 7 & x = -6 \\ x^3 & -6 < x < 1 \\ 1 & x = 1 \\ 2-x & x > 1 \end{cases}$$

$$\text{(a) } x = -6, \text{ (b) } x = 1$$

For problems 8 – 12 determine where the given function is discontinuous.

$$8. f(x) = \frac{x^2 - 9}{3x^2 + 2x - 8}$$

$$9. R(t) = \frac{8t}{t^2 - 9t - 1}$$

$$10. h(z) = \frac{1}{2 - 4\cos(3z)}$$

$$11. y(x) = \frac{x}{7 - e^{2x+3}}$$

$$12. g(x) = \tan(2x)$$

For problems 13 – 15 use the Intermediate Value Theorem to show that the given equation has at least one solution in the indicated interval. Note that you are NOT asked to find the solution only show that at least one must exist in the indicated interval.

$$13. 25 - 8x^2 - x^3 = 0 \text{ on } [-2, 4]$$

$$14. w^2 - 4\ln(5w + 2) = 0 \text{ on } [0, 4]$$

$$15. 4t + 10e^t - e^{2t} = 0 \text{ on } [1, 3]$$

Section 2-10 : The Definition of the Limit

Use the definition of the limit to prove the following limits.

1. $\lim_{x \rightarrow 3} x = 3$

2. $\lim_{x \rightarrow -1} (x + 7) = 6$

3. $\lim_{x \rightarrow 2} x^2 = 4$

4. $\lim_{x \rightarrow -3} (x^2 + 4x + 1) = -2$

5. $\lim_{x \rightarrow 1} \frac{1}{(x-1)^2} = \infty$

6. $\lim_{x \rightarrow 0^-} \frac{1}{x} = -\infty$

7. $\lim_{x \rightarrow \infty} \frac{1}{x^2} = 0$