

BIG IDEA:

Relating between f , f' and f''

$f(x)$	increasing	decreasing	concave up	concave down
$f'(x)$	positive (>0)	negative (<0)	increasing	decreasing
$f''(x)$	X	X	positive (>0)	negative (<0)

The diagram includes a vertical wavy line separating the first two columns. Above the $f(x)$ row, a circle with arrows indicates a local maximum (MAX) and a local minimum (MIN). To the right, a point is labeled 'P.o.I.' (Point of Inflection), with a '+' sign above it and a '-' sign below it, indicating the change in concavity.

Multiple Choice Practice

- Algebraically
- Graphically
- Numerically (Tables)
- Reasoning
- Calculator

Algebraically

1. Let f be the function given by $f(x) = 2xe^x$. The graph of f is concave down when

- (A) $x < -2$
- (B) $x > -2$
- (C) $x < -1$
- (D) $x > -1$
- (E) $x < 0$

2. The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for

- (A) $x < 0$
- (B) $x > 0$
- (C) $x < -2$ or $x > -\frac{2}{3}$
- (D) $x < \frac{2}{3}$ or $x > 2$
- (E) $\frac{2}{3} < x < 2$

3. Let f be the function with derivative given by $f'(x) = x^2 - \frac{2}{x}$. On which of the following intervals is f decreasing?

- (A) $(-\infty, -1]$ only
- (B) $(-\infty, 0)$
- (C) $[-1, 0)$ only
- (D) $(0, \sqrt[3]{2}]$
- (E) $[\sqrt[3]{2}, \infty)$

Key Takeaways:

4. What are all values of x for which the function f defined by $f(x) = x^3 + 3x^2 - 9x + 7$ is increasing?
- (A) $-3 < x < 1$
(B) $-1 < x < 1$
(C) $x < -3$ or $x > 1$
(D) $x < -1$ or $x > 3$
(E) All real numbers
5. What are all values of x for which the function f defined by $f(x) = (x^2 - 3)e^{-x}$ is increasing?
- (A) There are no such values of x
(B) $x < -1$ and $x > 3$
(C) $-3 < x < 1$
(D) $-1 < x < 3$
(E) All values of x
6. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = (x^2 - 4)g(x)$, which of the following is true?
- (A) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$
(B) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$
(C) f has relative minima at $x = -2$ and at $x = 2$
(D) f has relative maxima at $x = -2$ and $x = 2$
(E) It cannot be determined if f has an relative extrema

Key Takeaways:

7. Let f be a function with a second derivative given by $f''(x) = x^2(x - 3)(x - 6)$. What are the x -coordinates of the points of inflection of the graph of f ?

- (A) 0 only
- (B) 3 only
- (C) 0 and 6 only
- (D) 3 and 6 only
- (E) 0, 3, and 6

8. What is the x -coordinate of the point of inflection on the graph of $y = \frac{1}{3}x^3 + 5x^2 + 24$

- (A) 5
- (B) 0
- (C) $-\frac{10}{3}$
- (D) -5
- (E) -10

9. If $f''(x) = x(x + 1)(x - 2)^2$, then the graph of f has inflection points when $x =$

- (A) -1 only
- (B) 2 only
- (C) -1 and 0 only
- (D) -1 and 2 only
- (E) $-1, 0,$ and 2 only

Key Takeaways:

10. If f is the function defined by $f(x) = 3x^5 - 5x^4$, what are all the x -coordinates of points of inflections for the graph of f ?

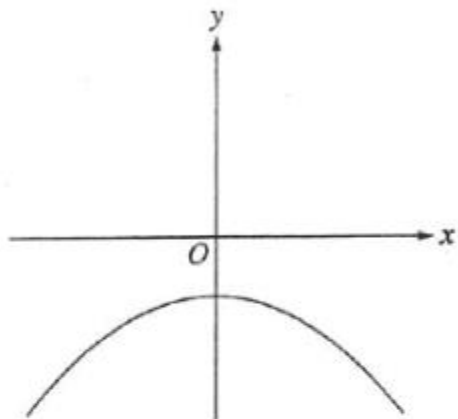
- (A) -1
- (B) 0
- (C) 1
- (D) 0 and 1
- (E) -1, 0, and 1

Key Takeaways:

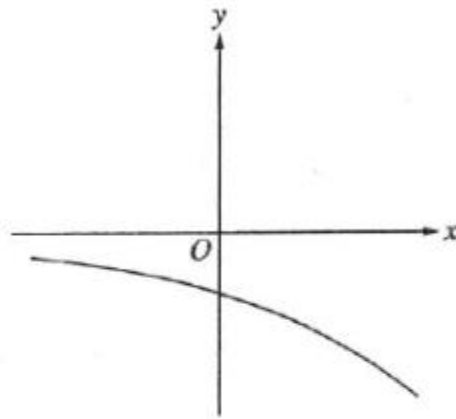
Graphically

11. The function f has the property that $f(x)$, $f'(x)$, and $f''(x)$ are negative for all real values x . Which of the following could be the graph of f ?

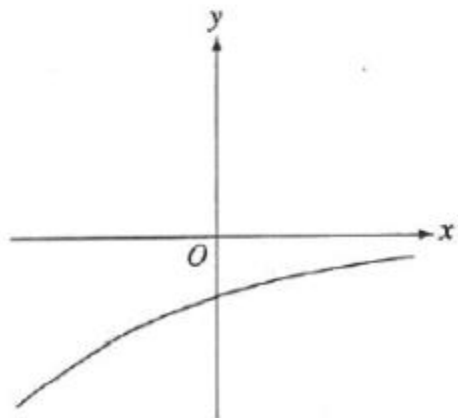
(A)



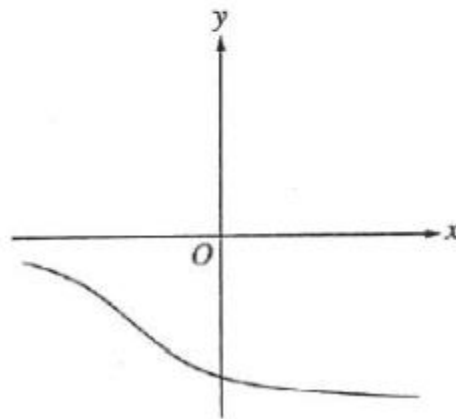
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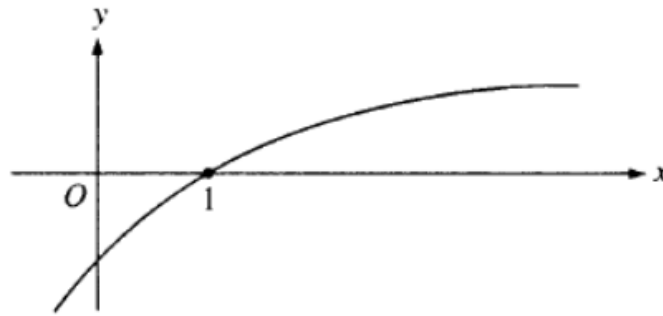
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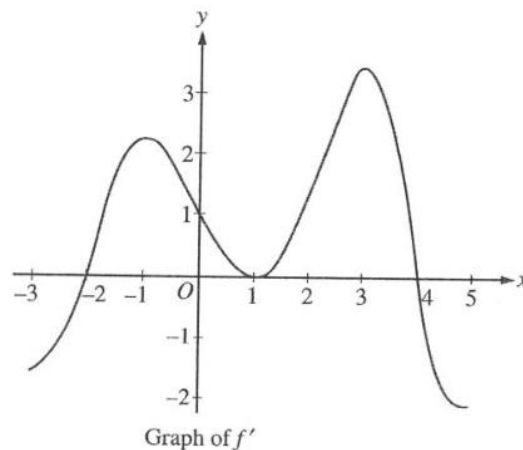


Key Takeaways:



12. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

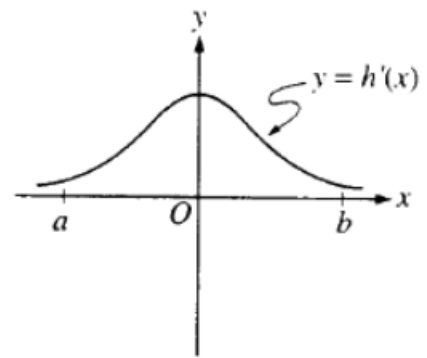
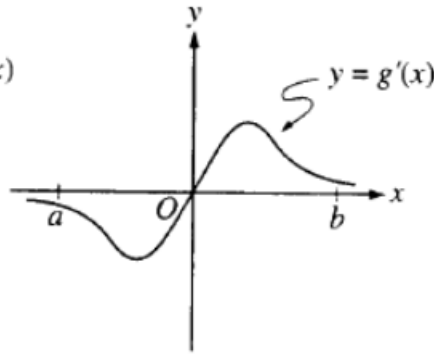
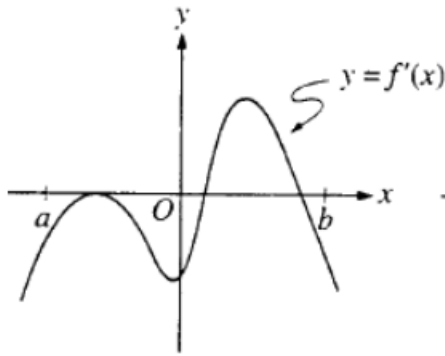
- (A) $f(1) < f'(1) < f''(1)$
- (B) $f(1) < f''(1) < f'(1)$
- (C) $f'(1) < f(1) < f''(1)$
- (D) $f''(1) < f(1) < f'(1)$
- (E) $f''(1) < f'(1) < f(1)$



13. The graph of the derivative of a function f is shown in the figure above. The graph has horizontal tangent lines at $x = -1$, $x = 1$, and $x = 3$. At which of the following values of x does f have a relative maximum?

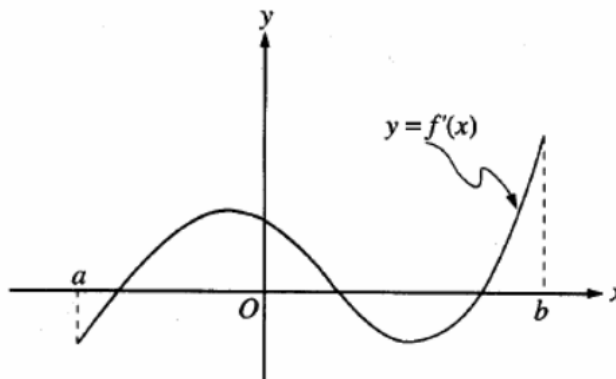
- (A) -2 only
- (B) 1 only
- (C) 4 only
- (D) -1 and 3 only
- (E) -2, 1, and 4

Key Takeaways:



14. The graphs of the derivatives of the functions f , g , and h are shown above. Which of the functions f , g , or h have a relative maximum on the open interval $a < x < b$?

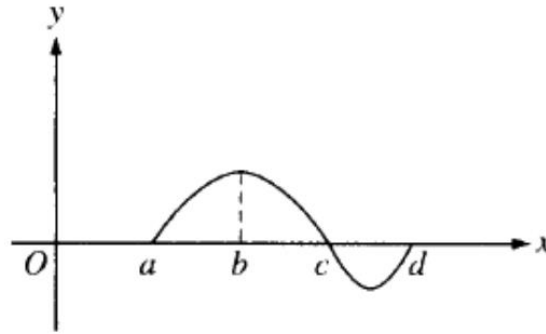
- (A) f only
- (B) g only
- (C) h only
- (D) f and g only
- (E) f , g , and h



15. The graph of f' , the derivative of f , is shown in the figure above. Which of the following describe all relative extrema of f on the open interval (a, b) ?

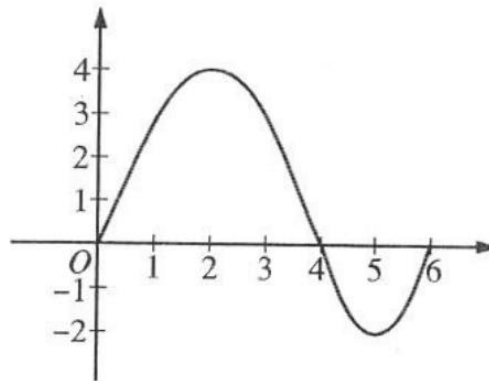
- (A) One relative maximum and two relative minima
- (B) Two relative maxima and one relative minimum
- (C) Three relative maxima and one relative minimum
- (D) One relative maximum and three relative minima
- (E) Three relative maxima and two relative minima

Key Takeaways:



16. The graph of f is shown in the figure above. If $g(x) = \int_a^x f(t) dt$, for what values of x does $g(x)$ have a maximum?

- (A) a
- (B) b
- (C) c
- (D) d
- (E) It cannot be determined from the information given.

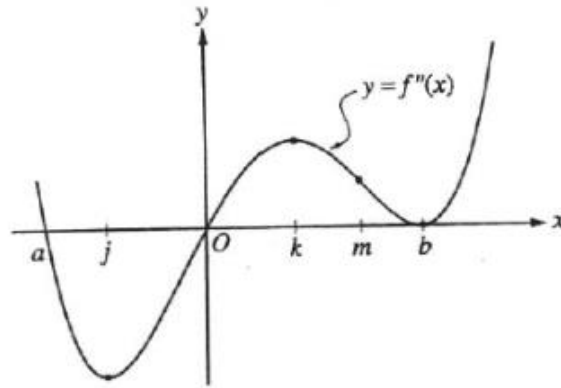


Graph of f

17. The graph of the function f shown above has horizontal tangents at $x = 2$ and $x = 5$. Let g be the function defined by $g(x) = \int_0^x f(t) dt$. For what values of x does the graph of g have a point of inflection?

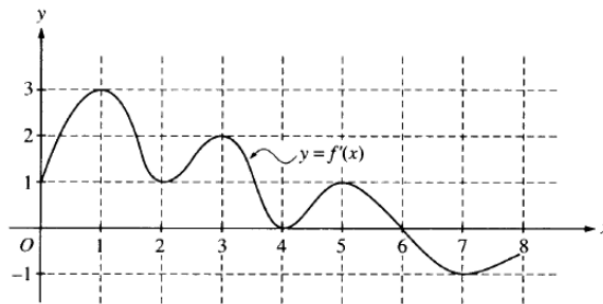
- (A) 2 only
- (B) 4 only
- (C) 2 and 5 only
- (D) 2, 4 and 5
- (E) 0, 4 and 6

Key Takeaways:



18. The second derivative of the function f is given by $f''(x) = x(x - a)(x - b)^2$. The graph of f'' is shown above. For what values of x does the graph of f have a point of inflection?

- (A) 0 and a only
- (B) 0 and m only
- (C) b and j only
- (D) 0, a , and b
- (E) b , j , and k



19. The function f defined on the closed interval $[0,8]$. The graph of its derivative f' is shown above. How many points of inflection does the graph of f have?

- (A) Two
- (B) Three
- (C) Four
- (D) Five
- (E) Six

Key Takeaways:

Numerically (Tables)

20. For all values of x , the continuous function f is positive and decreasing. Let g be the function given by $g(x) = \int_2^x f(t) dt$. Which of the following could be a table of values for g ?

(A)

x	$g(x)$
1	-2
2	0
3	1

(B)

x	$g(x)$
1	-2
2	0
3	3

(C)

x	$g(x)$
1	1
2	0
3	-2

(D)

x	$g(x)$
1	2
2	0
3	-1

(E)

x	$g(x)$
1	3
2	0
3	2

21. The function f is continuous on the closed interval $[2,4]$ and twice differentiable on the open interval $(2,4)$. If $f'(3) = 2$ and $f''(x) < 0$ on the open interval $(2,4)$, which of the following could be a table of values of f ?

(A)

x	$f(x)$
2	2.5
3	5
4	6.5

(B)

x	$f(x)$
2	2.5
3	5
4	7

(C)

x	$f(x)$
2	3
3	5
4	6.5

(D)

x	$f(x)$
2	3
3	5
4	7

(E)

x	$f(x)$
2	3.5
3	5
4	7.5

Key Takeaways:

x	-4	-3	-2	-1	0	1	2	3	4
$g'(x)$	2	3	0	-3	-2	-1	0	3	2

22. The derivative g' of a function g is continuous and has exactly two zeros. Selected values of g' are given in the table above. If the domain of g is the set of all real numbers, then g is decreasing on which of the following intervals?

- (A) $-2 \leq x \leq 2$ only
 (B) $-1 \leq x \leq 1$ only
 (C) $x \geq -2$
 (D) $x \geq 2$ only
 (E) $x \leq -2$ or $x \geq 2$

23. For all x in the closed interval $[2,5]$, the function f has a positive first derivative and a negative second derivative. Which of the following could be a table of values of f ?

(A)

x	$f(x)$
2	7
3	9
4	12
5	16

(B)

x	$f(x)$
2	7
3	11
4	14
5	16

(C)

x	$f(x)$
2	16
3	12
4	9
5	7

(D)

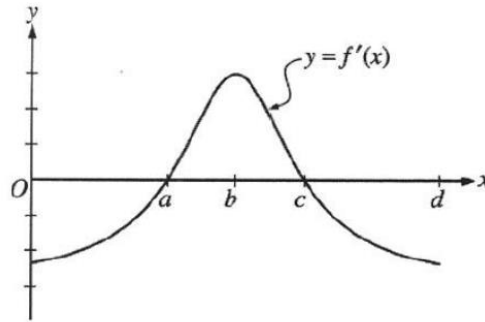
x	$f(x)$
2	16
3	14
4	11
5	7

(E)

x	$f(x)$
2	16
3	13
4	10
5	7

Key Takeaways:

Reasoning



24. The graph of f' , the derivative of a function f , is shown above. The domain of f is the open interval $0 < x < d$. Which of the following statements is true?
- A) f has a local minimum at $x = c$
 - B) f has a local maximum at $x = b$
 - C) The graph of f has a point of inflection at $(a, f(a))$
 - D) The graph of f has a point of inflection at $(b, f(b))$
 - E) The graph of f is concave up on the open interval (c, d)

x	0	1	2	3
$f(x)$	5	0	-7	4

25. The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?
- A) f is increasing on the interval $(0,2)$
 - B) f is decreasing on the interval $(0,2)$
 - C) f has a local maximum at $x = 1$
 - D) The graph of f has a point of inflection at $x = 1$
 - E) The graph of f change concavity in the interval $(0,2)$

Key Takeaways:

26. Let f be a twice-differentiable function with $g'(x) > 0$ and $g''(x) > 0$ for all real numbers x , such that $g(4) = 12$ and $g(5) = 18$. Of the following, which is a possible value for $g(6)$?

- (A) 15
- (B) 18
- (C) 21
- (D) 24
- (E) 27

Key Takeaways:

Calculator

27. [Calc] Let g be the function given by $g(x) = \int_0^x \sin(t^2) dt$ for $-1 \leq x \leq 3$. On which of the following intervals is g decreasing?

- (A) $-1 \leq x \leq 0$
- (B) $0 \leq x \leq 1.772$
- (C) $1.253 \leq x \leq 2.171$
- (D) $1.772 \leq x \leq 2.507$
- (E) $2.802 \leq x \leq 3$

28. [Calc] The graph of the function $y = x^3 + 6x^2 + 7x - 2 \cos x$ changes concavity at $x =$

- (A) -1.58
- (B) -1.63
- (C) 1.67
- (D) -1.89
- (E) -2.33

29. [Calc] Let f be the function given by $f(x) = \cos(2x) + \ln(3x)$. What is the least value of x at which the graph of f changes concavity?

- (A) 0.56
- (B) 0.93
- (C) 1.18
- (D) 2.38
- (E) 2.44

Key Takeaways:

30. [Calc] The first derivative of the function f is defined by $f'(x) = \sin(x^3 - x)$ for $0 \leq x \leq 2$. On what intervals is f increasing?
- (A) $1 \leq x \leq 1.445$ only
(B) $1 \leq x \leq 1.691$
(C) $1.445 \leq x \leq 1.875$
(D) $0.577 \leq x \leq 1.445$ and $1.875 \leq x \leq 2$
(E) $0 \leq x \leq 1$ and $1.691 \leq x \leq 2$
31. [Calc] Let f be the function given by $f(x) = \int_{1/3}^x \cos\left(\frac{1}{t^2}\right) dt$ for $\frac{1}{3} \leq x \leq 1$. At which of the following values of x does f attain a relative maximum?
- (A) 0.357 and 0.798
(B) 0.4 and 0.564
(C) 0.4 only
(D) 0.461
(E) 0.999
32. [Calc] Let f be the function with derivative given by $f'(x) = \sin(x^2 + 1)$. How many relative extrema does f have on the interval $2 < x < 4$?
- (A) One
(B) Two
(C) Three
(D) Four
(E) Five

Key Takeaways:

33. [Calc] If the derivative of f is given by $f'(x) = e^x - 3x^2$, at which of the following values of x does f have a relative maximum value?

- (A) -0.46
- (B) 0.20
- (C) 0.91
- (D) 0.95
- (E) 3.73

34. [Calc] The derivative of the function f is given by $f'(x) = x^2 \cos(x^2)$. How many points of inflection does the graph of f have on the open interval $(-2, 2)$?

- (A) One
- (B) Two
- (C) Three
- (D) Four
- (E) Five

Key Takeaways:

35. [Calc] The function f has first derivatives given by $f'(x) = \frac{\sqrt{x}}{1+x+x^3}$. What is the x -coordinate of the inflection point of the graph of f ?

- (A) 1.00
- (B) 0.473
- (C) 0
- (D) -0.278
- (E) The graph of f has no inflection point.

36. [Calc] Let f be the function with derivative defined by $f'(x) = \sin(x^3)$ on the interval $-1.8 < x < 1.8$. How many points of inflection does the graph of f have on this interval?

- (A) Two
- (B) Three
- (C) Four
- (D) Five
- (E) Six

Key Takeaways: